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ABSTRACT

In 1981 a nationwide assessment of achievement in writing geometry proofs was conducted by the Cognitive Development and Achievement in Secondary School Geometry project. Over 1,500 students in 11 schools in 5 states participated. This paper describes the sample, instruments, grading procedures, and selected results. Results include: (1) at the end of the year approximately half the students in courses teaching proof can do more than trivial proofs; and (2) there are no sex-related differences in proof-writing achievement. Relations between proof-writing achievement and both the van Hieles' levels of reasoning and achievement on standard content are also discussed. (Author/MP)

ACHIEVEMENT IN WRITING GEOMETRY PROOFS

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by

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Introduction

An understanding of the concept of proof and the facility to write proofs are fundamental to success in the study of higher mathematics. In the United States students usually first learn to write proofs in the high school geometry course, normally taken in the tenth grade. In spite of a long history of importance in the mathematics curriculum, writing geometry proofs is believed to be an area in which students experience little success (Gearhart, 1974). Yet the extent of students' difficulties with proofs and the degree to which their difficulties are due to any specific factor have been largely a matter of conjecture, for until now there has been little research on proof. No National Assessment data exist on proof-writing achievement, and neither the IEA nor NLSMA tested proof (Husen, 1967; NLSMA, 1968-72; Carpenter, 1978; NAEP, 1979). The research on proof conducted by classroom teachers, although rich in pedagogical advice, is generally methodologically weak (Fawcett, 1938; Smith, 1940; Ireland, 1973). In these studies the samples are small and specialized; the methods used to score the proofs are not described; and the reliability of the scoring systems is not reported.

The purpose of the research reported here was to gather base line data on proof-writing achievement from a large representative sample of high school geometry students in the United States. The central questions addressed by this study are:

1. To what extent can secondary school geometry students in the United States write geometric proofs like those given as theorems or exercises in commonly used texts?

2. To what extent is proof-writing achievement related to the student's van Hiele level of thinking, and his or her achievement on standard non-proof geometry content?

The van Hieles (van Hiele and van Hiele-Geldof, 1958; van Hiele, 1959; Freudenthal, 1973) hypothesized that students pass through a series of discrete levels when learning geometry. Each level is characterized by a particular type of geometric thinking. Specifically,

> at level 1, knowledge is obtained by observation;
> at level 2, knowledge is obtained from reasoning about properties of figures;
> at level 3, knowledge is derived via short chains of deductions about the relations between the properties of figures;
> at level 4, knowledge is derived by formal proofs in a deductive system, that is, by relating the relations derived at level 3;
> at level 5, knowledge is derived by reasoning about logical thinking itself, or by relating various deductive systems.

The van Hieles believed that students pass through these levels consecutively, and that certain instructional practices facilitate movement to higher levels. They also claimed that a student at a lower level cannot understand what is being discussed at a higher level. The van Hiele levels of geometric thought have been used to explain why many students have difficulty with geometry, particularly with geometric proofs. The course is thought by some to have been taught at a higher van Hiele level than most students have attained (Wirszup, 1976; Hoffer, 1981).

Sample

The sample for this study consisted of 1520 students from 74 geometry classes in 11 schools in 9 communities in 5 states. The states, California, Florida, Illinois, Massachusetts, and Michigan, are the home states of the members of the Advisory Board of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project. The communities and schools were chosen by the project staff as representative of educational and socio-economic conditions nationwide. Rural areas, small towns, medium cities, large cities, and suburbs are represented in the sample. Within each participating school only those classes that had been taught to write proofs, and whose teachers agreed to having their students tested on proof, participated in the study. In all, 17 honors, 38 average, and 19 heterogeneously grouped classes were tested. About two-thirds of the students were in grade 10. Mean age at the time of the spring testing was 16 years, 2 months. Over 95% of the students were 14-17 years of age. In these classes there were 741 (48.8%) females and 779 (51.2%) males. The percent of females and males was within one percent of the females and males in both national and school populations at ages 14-17. Most students were white, but there were sizeable black, hispanic, and oriental minorities.

Research Design

A standard pre- and post-test design was used. During the first week of the 1980-81 school year students took the Entering Geometry Student Test (EG) and the Van Hiele Geometry Test (VHF). Each of these multiple choice tests was developed by the CDASSG project staff. The EG is a 19 item, 25 minute test that measures achievement on geometry facts and concepts included in many junior high school curricula. The student's EG score is the number of items correct.

The VHF is a 35 minute, 25 item test with 5 subtests based on each of the 5 van Hiele levels. If a student answered correctly at least 4 of the 5 items in a given subtest, he or she was said to have mastered that level. The student's VHF score is the highest consecutive level he or she has mastered. Thus, the range of scores for VHF is 0 through 5. Students who mastered non-consecutive levels were not assigned van Hiele levels, and were excluded from the subsequent investigation of the relation between proof-writing achievement and van Hiele levels.

About one month before the end of the school year students took the Van Hiele Geometry Test again, and also a standardized geometry achievement test, and a CDASSG Proof Test. Following the same procedure used in the fall, a spring van Hiele level (VHS) was assigned to all students who mastered consecutive subtests on the Van Hiele Geometry Test. Achievement on standard non-proof content was measured by the 40 minute Comprehensive Assessment Program's (CAP) Geometry Test. The student's CAP score was the number of items correct from among the 40 multiple choice test items.

Three forms of the CDASSG Proof Test developed by this investigator were used in the study. Each form contains six items: two short answer items, and four full proofs for the student to write. One short answer item required students to fill in missing statements or reasons in a nearly complete proof; the other required the student to translate a sentence to an appropriate figure, and a symbolic representation of what is given and what is to be proved. The tests cover content that is common to the first two thirds of standard high school geometry texts, such as, congruent triangles, parallel lines, quadrilaterals, and similar triangles. Some items require proofs of standard textbook theorems; others are similar to exercises in texts. They represent a wide range of difficulty, and had been tested in two pilot studies. In each class the test forms were alternated among the students so that approximately one-third of each class took each form. The 35 minutes allotted was sufficient time for virtually all students to complete the test.

Items were graded on a scale of 0 to 4 based on criteria developed by Malone, et al. (1980). The criteria are:

0 - _noncommencement_ - no work or only meaningless work was done;
1 - _approach_ - some meaningful work was done, but an early impasse was reached;
2 - _substance_ - sufficient detail indicated that the student proceeded toward a rational solution, but major errors invalidate the proof;
3 - _results_ - minor errors flaw an otherwise valid proof;
4 - _completion_ - a complete valid proof was produced.

Eight experienced high school geometry teachers were hired as readers to grade the proof tests. General grading procedures were based on methods used to grade the Advanced Placement exams. Before grading each item the readers were trained to apply the general criteria to that item. Every item on each paper was graded independently by a different pair of readers who had no access to the student's name, school, grade, or sex, or the other reader's score. Inter-rater agreement averaged 86% across items and forms. Less than 2% of the scores of the pairs of readers differed by more than one point. When the readers' scores disagreed, a third independent blind reading was done, and the median score was chosen as the item score.

Two measures of proof-writing achievement are reported here. The first, called PRFTOT, is the sum of the six item scores, with a maximum possible score of 24. The second, called PRFCOR, is the number of full proofs on which the student scored at least 3 points, i.e., the number of proofs the student did "correctly." The maximum score for PRFCOR is 4.

Results

The selected results reported below have been grouped in two sections. First, key results from the assessment of proof-writing achievement are given. Data for achievement by item, overall achievement, achievement by school, and achievement by sex are included. Second, selected results of the investigation of the relation between proof-writing achievement and van Hiele levels and achievement on standard content are presented.

The Assessment of Proof-Writing Achievement

Figures 1 to 6 and Tables 1 to 6 illustrate the range of achievement by item. On the Figures the notation "item a-b" refers to item b on Form a of the Proof Test. Figures 1 and 2 were the two easiest items on the CDASSG Proof Tests; Figures 5 and 6 were the two most difficult; and Figures 3 and 4 were of intermediate difficulty. Among the results of the item analyses are:

<u>About 70% of the students can do simple proofs requiring only one deduction beyond those made from the given.</u>

Figures 1 and 2 show the two proof items that required the least number of deductions to complete the proof. Each required only one deduction beyond those made directly from the given. Approximately 72% of the students taking each item scored at least 3 of the 4 points possible on these items. Mean scores were approximately 3.0.

<u>Achievement is considerably lower on proofs requiring auxiliary lines or longer chains of reasoning.</u>

Among the ten more complex proofs the percent of students getting the item correct ranged from a high of 51% to a low of 6%. Mean scores on these items ranged from 2.25 to 0.77. Items shown in Figures 3 to 6 illustrate the range of achievement.

Within each form of the CDASSG Proof Tests the items are generally measuring a common underlying variable. The values of Cronbach's α are .86, .85, and .88 for Forms 1 to 3, respectively. A grouped frequency distribution of PRFTOT scores is given in Table 7. Mean PRFTOT scores were 12.19, 13.95 and 12.74 on Forms 1 to 3, respectively. Table 8 shows the distribution of PRFCOR scores. Mean achievement and the shape of the distributions differed significantly among the forms.[1] Thus, all subsequent references to overall proof-writing achievement are reported separately by form. Among the results of the analyses of overall proof-writing achievement are the following:

<u>After a full year of a geometry course with proof, only about half the students can do any more than simple proofs.</u>

Forms 1 and 2 contained the simple (some would say "trivial") proofs shown in Figures 1 and 2. Yet, as reported in Table 8, one fourth

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[1]Analyses of variance of PRFTOT vs Form and PRFCOR vs Form were both significant at the .001 level. An examination of the fit of hierarchical log-linear models to the frequency distributions of the PRFTOT and PRFCOR scores indicated a significant ($p < .001$) Form x Score interaction.

of the students taking Forms 1 and 2 solved no proofs correctly, and another fourth got only one proof correct. On Form 3 about 38% of the students failed to get a single proof correct, and again, only about half of the students got more than one proof correct.

Writing proofs is not an "all" or "nothing" task. Among the half of the population that can do more than simple proofs, there is a wide range of proof-writing achievement.

The percent of students who got either 2 or 3 proofs correct on any form, or 4 proofs correct on Forms 2 and 3, is nearly constant, ranging from 15% to 22% (Table 8). This result contradicts the belief held by some teachers that proofs are something students either "get" or "don't get." Only on Form 1, which contained the most difficult item of all (Figure 6 and Table 6) is there an exception to this pattern.

Analyses of proof-writing achievement by school and sex show that:

There are strong school effects on proof-writing achievement.

On Form 3 school means for PRFTOT ranged from 5.76 to 16.85 of the 24 possible points (Table 9). The three lowest school means were each less than half the highest school mean. The mean number of full proofs correct on Form 3 ranged from 0.33 to 2.52 of the 4 possible proofs. The three lowest schools averaged less than one proof correct per pupil; the best schools averaged more than two proofs correct per pupil. Similar results occurred on the other forms. School means for PRFTOT ranged from 5.59 to 15.27 on Form 1, and from 5.77 to 17.05 on Form 2. Mean number of proofs correct (PRFCOR) by school ranged from 0.45 to 1.91 on Form 1, and from 0.45 to 2.42 on Form 2.

There are no consistent sex-related differences in proof-writing achievement.

Mean PRFTOT scores are higher for males on the first two forms, and for the females on Form 3 (Table 10). The largest mean difference is 0.79 points on a 24 point test. None of these differences is significant at the .05 level. The mean PRFCOR scores favor the boys by 0.02, 0.26, and 0.11 points on Forms 1 to 3, respectively. The difference in means on Form 2 is statistically significant ($p < .05$), but the difference represents less than 20% of the standard deviation of the PRFCOR distribution, and thus, a rather small educational difference. When mean proof-writing achievement by sex is adjusted for differences in entering knowledge of geometry (EG), the adjusted means for both PRFTOT and PRFCOR on all three forms favor the girls. The implications of this result are discussed in greater detail by Senk and Usiskin (in preparation).

Relations Between Proof-Writing Achievement And Selected Factors

In examining the relations between proof-writing, van Hiele levels, and achievement on standard non-proof content, only data from those 1130 students who took all five tests were used. Of these 1130 students, 751 (66.5%) fit the van Hiele model in both the fall and spring. These 751 students (n = 248, 241, and 262 for Forms 1 to 3, respectively) comprise the sample upon which the results of this section are based. There were no significant differences in the EG, CAP, or Proof Test scores between the 1520 students who took the Proof Tests and the subset taking all five tests who fit the van Hiele model.

Conclusions of this investigation include:

<u>Both van Hiele level and achievement on standard content correlate highly and significantly with proof-writing achievement.</u>

Table 11 shows that the correlation between the proof-writing variables (PRFTOT and PRFCOR) is approximately

- .4 to .5 with fall van Hiele level,
- .5 to .6 with spring van Hiele level,
- .5 to .6 with entering geometry knowledge, and
- .6 with concomitant standardized achievement on non-proof content.

All these correlations are significant at the .0001 level.

<u>Fall van Hiele level, as a single predictor, sorts students into strikingly different categories of geometry proof writers.</u>

If the criterion for success in writing geometry proofs is defined to be getting at least 3 of 4 full proofs correct, then students entering the course at van Hiele level 0 or 1 have less than one chance in three of achieving success; those entering at level 2 have about a 50% chance of success; and those entering at levels 3 or 4 have far greater than a 50% chance of success (Table 12).

Mean PRFTOT scores by fall van Hiele level show a strong linear trend (Table 13). Students who started the year at level 0, on the average, earned between 8 and 10 points on the 24 point Proof Tests. For each consecutive van Hiele level the mean PRFTOT scores are 3 to 4 points higher. Students who started the year at levels 3 or 4 had average PRFTOT scores of 19 to 22 points.

An analysis of variance with post hoc contrasts confirms the statistical significance of the relation between PRFTOT and VHF ($p < .0001$). In general, students entering the course at van Hiele level 1 did significantly better ($p < .0121$, $p < .0001$, $p < .0001$), than those who entered at level 0. Students who started the year at level 3 did significantly better ($p < .0134$, $p < .0050$) than those who started at level 2 on Forms 1 and 2 of the Proof Test. But on Form 3 the mean performance of those who started at level 3 was not significantly different than the performance of those who started at level 2 (Table 14).

<u>Overall, fall van Hiele level and entering knowledge of geometry account for 30% to 40% of the variance in proof-writing achievement.</u>

VHF accounts for between 18% and 25% of the variance in PRFTOT scores. EG accounts for an additional 10% to 16%. Together they account for 31%, 41%, and 29% of the variance in PRFTOT scores on Forms 1 to 3, respectively.

<u>Spring van Hiele level sorts students into equally striking categories of geometry proof writers.</u>

Tables 15 and 16 illustrate the highly significant ($p < .0001$) relation between VHS and PRFCOR and PRFTOT, respectively. Those students who were at levels 0, 1 or 2 in the spring (and more than 60% of the sample was still at these levels at the end of a full year of geometry) were generally unsuccessful at proof. Students at level 3 earned higher PRFTOT scores than average, but only about half of them could do at least three of the four full proofs correctly. However, students at levels 4 or 5 not only scored considerably above the mean in total proof score; they also were quite successful on the full proofs.

<u>Spring van Hiele level and concomitant achievement on a standardized geometry test account for more than 55% of the variance in proof-writing achievement.</u>

VHS accounts for between 35% and 41% of the variance in PRFTOT scores. CAP accounts for an additional 17% to 22%. In all, VHS and CAP account for 59%, 58%, and 56% of the variance in PRFTOT scores on Forms 1 to 3, respectively.

Conclusions

The primary purpose of this research was to gather data on achievement in writing geometry proofs from a large representative sample of high school geometry students in the United States. Until this study no such data existed. The second purpose of this study was to examine selected factors that were thought to be related to proof-writing achievement.

The assessment portion of this study found that only half the students in geometry courses teaching proof were able to get more than one of four proofs correct, and that about one fourth of the students could not even write a simple proof. There are very large differences in proof-writing achievement by school, but no sex-related differences in proof-writing achievement.

This study also found that achievement in writing proofs is strongly related to the student's van Hiele level in both the fall and spring. Wirszup's (1976) assertion that most students enter the high school geometry course at levels 0 and 1 is supported by data from the CDASSG project. Furthermore, most students who start the year at such low levels do not succeed in learning to write proofs. The van Hieles' (van Hiele and van Hiele-Geldof, 1958; van Hiele, 1959) hypotheses that only at level 3 do students begin to understand deductive proof, and that only students who have reached levels 4 or 5 are able to write their own proofs, are generally supported by the data from this research. Students who enter the course at level 2 have about a 50% chance of being successful at writing proofs.

However, the limitations of the reliability of the instrument and scoring system used to measure van Hiele levels should be kept in mind when interpreting these results. First, the van Hiele Geometry Test consists of five subtests with only five multiple choice items in each subtest. The least bit of inattention on the part of the student could influence the measure of his or her van Hiele level. Second, the CDASSG staff has some doubts about how the highest van Hiele level is defined, and if it is possible to measure this level with a multiple choice format. In fact, while visiting the United States in 1980, Pierre van Hiele said he no longer believed in a level 5. Lastly, when mastery of a van Hiele level is redefined, in an operational sense, to mean getting three of five questions in a subtest correct, rather than four of five as used here, the relation between achievement on proof and van Hiele levels changes slightly. (Zalman Usiskin will talk more about this issue at his session later this week.) Thus, although the van Hiele model of five discrete levels of reasoning is a significant predictor of proof-writing achievement, it requires further study. Furthermore, because the van Hiele levels are highly correlated with achievement on the tests of standard content (EG and CAP), one can question the degree to which the van Hiele level is any different from an achievement test score.

In spite of these limitations, this study makes a useful contribution to the educational research community. First, it shows that many of the difficulties associated with testing proof can be overcome. Criteria for grading proofs written by students who have studied from different texts and teachers with different systems of notation have been developed. They have been applied consistently by experienced geometry teachers.

These procedures can, no doubt, be applied to measuring other types of problem-solving on a large scale.

Second, this research provides the mathematics education community with data that serve as a base line for future research, evaluation and curriculum development. On the one hand, these data reveal a very low level of achievement in writing geometry proofs. On the other hand, they show that some schools are quite successful at teaching proof. One cannot help but wonder how we might improve achievement in the unsuccessful schools. Moreover, the fact that girls and boys did equally well at writing geometry proofs, forces us to examine other studies of mathematics achievement by sex, and to ask how learning to write proofs may be different than learning other complex tasks that for years have favored males.

Lastly, the analysis of the relations between proof-writing achievement and van Hiele levels has identified a variable that is worthy of further research. It appears from this study that level 2 is a critical level for predicting success in writing geometry proofs. Students entering the course at level 2 or above have at least a 50% chance of success. But might even students who enter the course at levels 0 and 1 be able to learn to write proofs if given suitable instruction? If so, what might such an optimal instructional program include? Data gathered by the CDASSG project, but not yet analyzed, deal with such issues as tracking, content-covered and other aspects of opportunity to learn. We hope to say more about the relations between van Hiele levels, opportunity to learn and achievement in writing proofs in the near future.

FIGURE 1: Item 1-3

Write this proof in the space provided.

GIVEN: M is the midpoint of $\overline{AB}$.

M is the midpoint of $\overline{CD}$.

PROVE: $\triangle ACM \cong \triangle BDM$

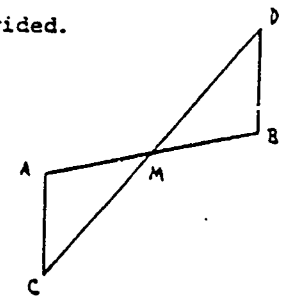


TABLE 1: Distribution of Scores on Item 1-3

	0	1	2	3	4
n	87	38	18	44	319
%	17.2	7.5	3.6	8.7	63.0
cum. %	17.2	24.7	28.3	37.0	100.0
N = 506	mean score = 2.93			s.d. = 1.57	

FIGURE 2: Item 2-3

Write this proof in the space provided.

GIVEN: $\overline{BD} \cong \overline{EC}$

$\angle 1 \cong \angle 2$

$\angle B \cong \angle E$

PROVE: $\overline{AB} \cong \overline{EF}$

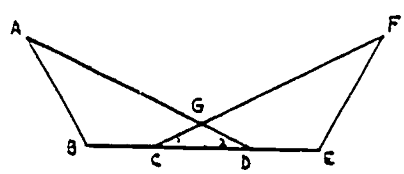


TABLE 2: Distribution of Scores on Item 2-3

	0	1	2	3	4
n	97	17	26	27	341
%	19.1	3.3	5.1	5.3	67.1
cum.%	19.1	22.4	27.5	32.8	100.0
N = 508	mean score = 2.98			s.d. = 1.61	

FIGURE 3: Item 3-4

Write this proof in the space provided.

GIVEN: Quadrilateral HIJK

HI = HK

IJ = JK

PROVE: $\angle I \cong \angle K$

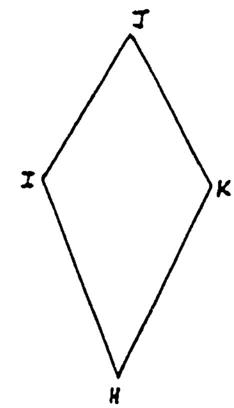


TABLE 3: Distribution of Scores on Item 3-4

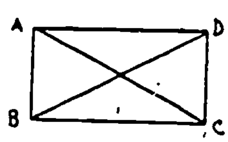
	0	1	2	3	4
n	201	25	21	19	240
%	39.7	4.9	4.2	3.8	47.4
cum. %	39.7	44.6	48.8	52.6	100.0
N = 506	mean score = 2.14		s.d. = 1.89		

FIGURE 4: Item 3-5

Here is a theorem you have had. Complete its proof in the space provided.

Theorem: The diagonals of a rectangle are congruent.

FIGURE:



GIVEN: ABCD is a rectangle.

TO PROVE: $\overline{AC} \cong \overline{BD}$

PROOF:

TABLE 4: Distribution of Scores on Item 3-5

	0	1	2	3	4
n	185	97	63	40	121
%	36.6	19.2	12.5	7.9	23.9
cum. %	36.6	55.8	68.3	76.2	100.0
N = 506	mean score = 1.63		s.d. = 1.60		

FIGURE 5: Item 2-6

Write this proof in the space provided.

GIVEN: $\triangle ABF \sim \triangle ACE$

$\triangle FDE \sim \triangle ACE$

PROVE: BCDF is a parallelogram.

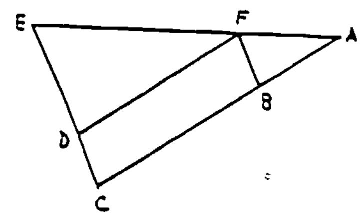
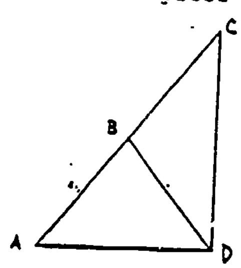


TABLE 5: Distribution of Scores on Item 2-6

	0	1	2	3	4
n	289	93	34	26	66
%	56.9	18.3	6.7	5.1	13.0
cum. %	56.9	75.2	81.9	87.0	100.0
N = 508	mean score = 0.99			s.d. = 1.42	

FIGURE 6: Item 1-6

Write this proof in the space provided.



GIVEN: B is the midpoint of $\overline{AC}$.

AB = BD.

PROVE: $\angle CDA$ is a right angle.

TABLE 6: Distribution of Scores on Item 1-6

	0	1	2	3	4
n	236	206	34	4	26
%	46.6	40.7	6.7	0.8	5.1
cum. %	46.6	87.3	94.0	94.8	100.0
N = 506	mean score = 0.77			s.d. = 0.99	

TABLE 7

NUMBER (n), PERCENT (%), AND CUMULATIVE PERCENT (cum.%) OF STUDENTS WITH GIVEN PRFTOT SCORES, BY FORM

PRFTOT Interval	Form 1			Form 2			Form 3		
	n	%	cum.%	n	%	cum.%	n	%	cum.%
0 - 4	93	18.4	18.4	49	9.6	9.6	105	20.8	20.8
5 - 8	60	11.9	30.2	68	13.4	23.0	70	13.8	34.6
9 -12	77	15.2	45.4	83	16.3	39.3	70	13.8	48.4
13-16	116	22.9	68.3	99	19.5	58.9	59	11.7	60.0
17-20	119	23.5	81.9	111	21.9	80.8	92	18.2	78.2
21-24	41	8.1	100.0	98	19.2	100.0	110	21.7	100.0
Total	506			508			506		
Mean PRFTOT Score ± s.d.	12.19±6.48			13.95±6.50			12.74±7.59		

TABLE 8

NUMBER (n), PERCENT (%), AND CUMULATIVE PERCENT (cum.%) OF STUDENTS GETTING THE FULL PROOFS CORRECT, BY FORM

Number of Full Proofs Correct	Form 1			Form 2			Form 3		
	n	%	cum.%	n	%	cum.%	n	%	cum.%
0	132	26.1	-	124	24.4	-	192	37.9	-
1	145	28.7	54.8	114	22.4	46.8	65	12.8	50.7
2	111	21.9	76.7	88	17.3	64.1	76	15.0	65.7
3	99	19.6	96.3	104	20.5	84.6	78	15.4	81.1
4	19	3.8	100.0	78	15.4	100.0	95	18.8	100.0
Total	506			508			506		
Mean Number of Full Proofs Correct ± s.d.	1.46±1.18			1.80±1.41			1.64±1.56		

TABLE 9

MEAN PROOF-WRITING ACHIEVEMENT BY SCHOOL ON FORM 3

School	n	PRFTOT		PRFCOR	
		mean	s.d.	mean	s.d.
1	21	5.76	4.53	0.33	0.97
2	80	8.26	6.36	0.74	1.14
3	18	13.67	5.06	1.44	1.25
4	12	14.58	6.46	1.75	1.48
5	27	13.00	6.59	1.74	1.35
6	45	12.89	7.13	1.62	1.48
7	153	16.85	6.87	2.52	1.47
8	23	16.17	5.55	2.22	1.44
9	41	14.61	6.72	1.93	1.57
10	21	11.86	7.34	1.48	1.36
11	65	7.83	6.95	0.75	6.95
Total	506	12.74	7.59	1.64	1.56

TABLE 10

MEAN PROOF-WRITING ACHIEVEMENT BY SEX AND FORM

		Females			Males		
	Form	n	mean	s.d.	n	mean	s.d.
PRFTOT	1	240	12.06	6.50	266	12.30	6.47
	2	240	13.53	6.39	268	14.32	6.58
	3	261	12.85	7.44	245	12.63	7.75
PRFCOR	1	240	1.45	1.17	266	1.47	1.19
	2	240	1.66	1.35	268	1.92	1.45
	3	261	1.59	1.53	245	1.70	1.59

TABLE 11

CORRELATION COEFFICIENTS, BY FORM

	VHF	VHS	CAP	PRFTOT	PRFCOR	
EG	.62	.66	.61	.54	.53	Form 1
	.58	.57	.62	.61	.61	Form 2
	.52	.56	.61	.51	.47	Form 3
VHF		.58	.48	.46	.48	
		.52	.57	.50	.51	
		.51	.49	.43	.37	
VHS			.69	.63	.62	
			.63	.57	.56	
			.60	.59	.53	
CAP				.73	.70	
				.73	.71	
				.69	.68	
PRFTOT					.90	
					.94	
					.94	

Notes: 1. EG = raw score on Entering Geometry Student Test,
VHF = Van Hiele Level in Fall,
VHS = Van Hiele Level in Spring,
CAP = raw score on Comprehensive Assessment Program's Geometry Test,
PRFTOT = raw score on CDASSG Proof Test,
PRFCOR = number of full proofs correct.

2. Throughout the table the upper correlation coefficient refers to Form 1, the middle coefricient to Form 2, and the lower coefficient to Form 3.

3. On Form 1, n = 248; on Form 2, n = 241; on Form 3, n = 262.

4. All correlations are significant at the .0001 level.

TABLE 12

TOTAL NUMBER (N) OF STUDENTS AT A GIVEN VAN HIELE LEVEL IN FALL, AND NUMBER (n) AND PERCENT (%) GETTING AT LEAST 3 FULL PROOFS CORRECT, BY FORM

	Form 1			Form 2			Form 3		
VHF	N	n	%	N	n	%	N	n	%
0	59	2	3	66	6	9	75	11	16
1	117	19	16	123	37	31	130	43	33
2	51	22	43	36	20	56	44	23	52
3	21	12	57	15	15	100	11	8	73
4	0	-	-	1	1	100	2	1	50
Total	248	55	22	241	79	33	262	86	33

TABLE 13

MEAN PRFTOT SCORES BY FALL VAN HIELE LEVEL AND TEST FORM

VHF	Form 1			Form 2			Form 3		
	n	mean	s.e.	n	mean	s.e.	n	mean	s.e.
0	59	8.89	0.76	66	9.70	0.71	75	8.12	0.79
1	117	11.23	0.56	123	13.97	0.52	130	12.29	0.66
2	51	15.25	0.76	36	17.00	0.90	44	16.39	0.94
3	21	18.90	0.76	15	21.87	0.52	11	20.27	1.41
4	0	-	-	1	24.00	-	2	19.50	4.50
Total	248	12.17	0.40	241	13.78	0.41	262	12.18	0.48

TABLE 14

ANALYSIS OF VARIANCE OF PROOF-WRITING ACHIEVEMENT (PRFTOT) VS. FALL VAN HIELE LEVEL (VHF), BY FORM

Form 1

Source	df	Sum of Squares	Mean Square	F	p
VHF	3	2153.11	717.70	22.46	.0001
contrast					
μ_2 vs μ_3	1	198.16			.0134
μ_0 vs μ_1	1	204.18			.0121
$\mu_0+\mu_1$ vs $\mu_2+\mu_3$	1	1750.77			.0001
ERROR	244	7797.11	31.96		
TOTAL	247	9950.22		R^2 = .22	

Form 2*

Source	df	Sum of Squares	Mean Square	F	p
VHF	3	2458.44	819.48	26.23	.0001
contrast					
μ_2 vs μ_3	1	250.78			.0050
μ_0 vs μ_1	1	783.33			.0001
$\mu_0+\mu_1$ vs $\mu_2+\mu_3$	1	1424.33			.0001
ERROR	236	7373.54	31.24		
TOTAL	239	9831.98		R^2 = .25	

Form 3*

Source	df	Sum of Squares	Mean Square	F	p
VHF	3	2735.88	911.96	18.52	.0001
contrast					
μ_2 vs μ_3	1	132.91			.1016
μ_0 vs μ_1	1	827.95			.0001
$\mu_0+\mu_1$ vs $\mu_2+\mu_3$	1	1775.02			.0001
ERROR	256	12603.43	49.23		
TOTAL	259	15339.30		R^2 = .18	

*The three students at level 4, one taking Form 2 and two taking Form 3, were not included in these analyses.

TABLE 15

TOTAL NUMBER (N) OF STUDENTS AT A GIVEN VAN HIELE LEVEL IN SPRING, AND NUMBER (n) AND PERCENT (%) GETTING AT LEAST 3 FULL PROOFS CORRECT, BY FORM

	Form 1			Form 2			Form 3		
VHS	N	n	%	N	n	%	N	n	%
0	29	1	3	28	1	4	33	1	3
1	55	3	5	45	6	13	61	9	15
2	82	17	21	83	18	22	78	21	27
3	56	18	32	68	39	57	74	41	55
4	17	9	53	13	11	85	9	7	78
5	9	7	78	4	4	100	7	7	100
Total	248	55	22	241	79	33	262	86	33

TABLE 16

MEAN PROFTOT SCORES BY SPRING VAN HIELE LEVEL AND TEST FORM

VHS	Form 1			Form 2			Form 3		
	n	mean	s.e.	n	mean	s.e.	n	mean	s.e.
0	29	6.00	0.91	28	7.96	1.05	33	5.88	0.91
1	55	7.91	0.70	45	10.98	0.82	61	7.79	0.85
2	82	12.45	0.63	83	12.46	0.59	78	11.67	0.83
3	56	15.95	0.55	68	17.99	0.56	74	17.30	0.62
4	17	18.53	0.64	13	19.76	1.80	9	20.56	1.25
5	9	19.78	0.68	4	22.75	0.95	7	20.86	0.70
Total	248	12.17	0.40	241	13.78	0.41	262	12.18	0.48

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